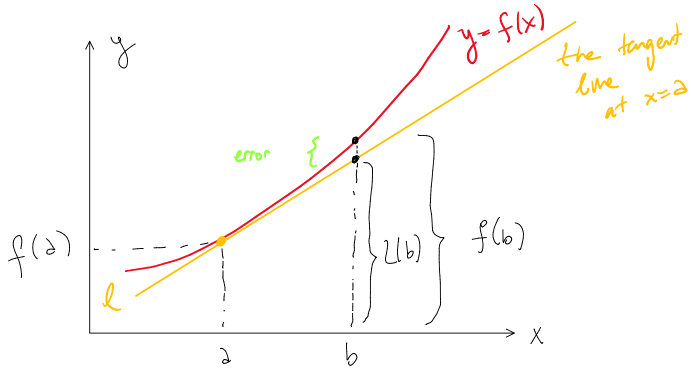
Section 4.9

Monday, March 30, 2020 4:33 PM



The equation of ℓ is

$$y - f(a) = f'(a)(x-a)$$

$$y = f'(a)(x-a) + f(a)$$

Let f be a function which is differentiable at $x=a$.

If a point b is "close to" a, then $f(b)$ would be "close to" $L(b)$, where $L(x)$ is the function, whose graph is the tangent line of f at $x=a$, given by

$$L(x) = f'(a)\cdot(x-a) + f(a)$$

which is called **the linearization of f at $x=a$.**

$$\boxed{\text{If } x \approx a, \text{ then } L(x) \approx f(x)}$$

Example: Find an approximate value of $\sqrt{51}$ using linearization.

Solution: We will use the linearization of $f(x) = \sqrt{x}$ at $x=49$. We have $f'(x) = \frac{1}{2\sqrt{x}}$. So

$$L(x) = f'(49)\cdot(x-49) + f(49) = \frac{1}{2\sqrt{49}}\cdot(x-49) + \sqrt{49} = \frac{1}{14}(x-49)+7$$

$$\sqrt{51} = f(51) \approx L(51) = \frac{1}{14}(51-49)+7 = \frac{\cancel{2}}{\cancel{14}_{7}}+7 = \frac{50}{7}$$